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Engineering Notes

Uncertainty Quantification in Flutter Analysis for an Airfoil with Preloaded Free Play

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Nomenclature

b	=	semichord
C_h	=	damping coefficient in plunge
C_α	=	damping coefficients in pitching
h	=	plunge deflection
I_α	=	moment of inertia about elastic axis
K_α	=	linear stiffness coefficients in pitching
K_ξ	=	linear stiffness coefficients in plunge
m	=	mass
S	=	static moment about the elastic axis
U	=	velocity
U^*	=	nondimensional velocity
α	=	pitching angle
τ	=	nondimensional time
ω_α	=	natural frequencies in pitching modes
ω_ξ	=	natural frequencies in plunge modes

Introduction

IN CLASSIC aeroelasticity, the instability of flutter has been studied by a very deterministic method with the intent to avoid catastrophic aircraft destruction. In this analytical method the attributes and tests for items such as mass, stiffness, inertias, and dimension of the aircraft configurations are assumed to be the best estimation and deterministic values. In fact, however, these values are variable from one aircraft to another, due to the differences of manufacturing run and operational condition. Recently, measurements of component weight and hinge line variation were conducted on a small sample size of 24 samples [1]. It shows that the weight variation is about 5% and hinge line inertia is about 20%. These variations are in fact parametric uncertainties. As a consequence, in order to obtain the more reliable aeroelastic stability of system, we should take the effect of parametric uncertainties into the consideration. Parametric uncertainties owe their origin to many sources, which include [2] 1) stochastic variations in material properties, 2) stochastic variations in structural dimensions, 3) stochastic variations in boundary conditions due to preload and relaxation variations in mechanical joints, and 4) stochastic variations of external excitations.

Military aircraft require flight tests throughout their useful life as their operational demands changes and new external stores, but the conflict of increasingly constrained budgets and expanding requirements for performance induce designers to renovate our approach to design for and demonstration of aeroelastic stability. Recently the U.S. Air Force Office of Scientific Research and the U.S. Air Force Research Laboratory organized a workshop, which included the role of uncertainty quantification (UQ) in efforts to understand the physics of nonlinear aeroelasticity and to certify aeroelastic stability [2]. The participants of the workshop developed a strong consensus that UQ must play a prominent role in the future of aeroelasticity research.

Civil engineers have been involved in studying the influence of uncertainties of structural properties (in particular, damping and velocity) on the reliability analysis of flutter of a bridge and other buildings [3–5]. Civil engineers are interested in determining a probability of the bridge or a structure failure due to flutter for a given period.

To figure out the effect of uncertain parameters many researchers have done highly effective jobs. The impact of structural and material uncertainties on the flutter characteristics of plate and shell has been studied by Liaw and Yang [6,7] and Lindsley et al. [8,9]. Castravete and Ibrahim [2] researched the effect of stiffness uncertainties on the flutter of a cantilever wing using a stochastic finite element approach. Kurdi et al. [10] studied the uncertainty effects on the Goland wing. Heeg [11] applied stochastic treatment to wind tunnel data to predict flutter margins. Pettit [12] gave a good overview and review about uncertainties in aeroelasticity.

Recently, the influence of parameter uncertainties on the response of a typical airfoil section was considered by a few researchers [13,14]. In 2003, Pettit and Beran [15] researched the impact of parametric uncertainties in cubic nonlinear twist stiffness on the response of the airfoil with pitch and plunge degree of freedom.

In this paper, we research the effect of uncertainties in free play with preload on flutter characteristics of the airfoil section. Thus, we employ a two-degree-of-freedom airfoil with preloaded free play. The use of such an elementary model is computationally expedient, and our goals are to figure out the impact of parametric uncertainties in free play with preload on the limit cycle oscillation LCO of the airfoil and to demonstrate the application of standard probability concepts and Monte Carlo simulation (MCS) to quantify the uncertainties in aeroelastic system, at last, to provide a simple example of how probabilistic aeroelastic analyses might be performed.

Deterministic Model of Airfoil Oscillating in Pitch and Plunge

Two-Degree-of-Freedom Airfoil Motion with Nonlinear Stiffness

Figure 1 shows that a two-degree-of-freedom airfoil oscillates in pitching and plunge [16]. The plunge deflection is denoted by h, positive in the downward direction, and α is the pitching angle about the elastic axis, positive nose-up. The elastic axis is located at a distance $a_h b$ from the midchord, while the mass center is located at a distance $x_a b$ from the elastic axis. Both distances are positive when measured toward the trailing edge of the airfoil. The aeroelastic equations of motion for this two-degree-of-freedom airfoil with nonlinear stiffness can be written in nondimensional form [16]:

$$\ddot{\xi} + x_\alpha \ddot{\alpha} + 2\zeta_\xi \frac{\bar{\omega}}{U^*}\dot{\xi} + \left(\frac{\bar{\omega}}{U^*}\right)^2 G(\xi) = -\frac{1}{\pi\mu} C_L(\tau) + \frac{P(\tau) b}{m U^2} \quad (1)$$

$$\frac{x_\alpha}{r_\alpha^2}\ddot{\xi} + \ddot{\alpha} + 2\frac{\zeta_\alpha}{U^*}\dot{\alpha} + \frac{1}{U^{*2}} M(\alpha) = \frac{2}{\pi\mu r_\alpha^2} C_M(\tau) + \frac{Q(\tau)}{m U^2 r_\alpha^2} \quad (2)$$

where $\xi = h/b$, $K_\xi = K_h$, $x_\alpha = S/bm$, $\omega_\xi = (K_\xi/m)^{1/2}$, $\omega_\alpha = (K_\alpha/I_\alpha)^{1/2}$, $r_\alpha = (I_b/mb^2)^{1/2}$, $\zeta_\xi = C_h/2(mK_h)^{1/2}$, $\zeta_\alpha = C_\alpha/2(I_\alpha K_\alpha)^{1/2}$,

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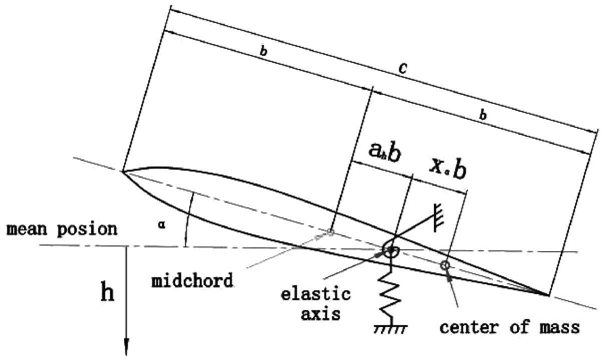

Fig. 1 Two-degree-of-freedom airfoil motion.

$G(\xi) = \bar{G}(h)/K_\xi$, $M(\alpha) = \bar{M}(\alpha)/K_\alpha$, $\bar{G}(h)$ and $\bar{M}(\alpha)$ are the nonlinear stiffness in plunge and pitching, and K_α and K_ξ are the corresponding linear stiffness coefficients in plunge and pitching. In Eqs. (1) and (2), U^* is a nondimensional velocity defined as

$$U^* = U/b\omega_\alpha$$

and $\bar{\omega}$ is given by

$$\bar{\omega} = \frac{\omega_\xi}{\omega_\alpha}$$

where ω_ξ and ω_α are the uncoupled natural frequencies in plunge and pitching modes, respectively, and a dot accent denotes differentiation with respect to nondimensional time τ, which is defined as

$$\tau = \frac{Ut}{b}$$

For incompressible flow, Fung [17] gives the following expressions for $C_L(\tau)$ and $C_M(\tau)$:

$$\begin{aligned} C_L(\tau) &= \pi(\ddot{\xi} - a_h\ddot{\alpha} + \dot{\alpha}) + 2\pi[\alpha(0) + \dot{\xi}(0) \\ &\quad + (0.5 - a_h)\dot{\alpha}(0)]\phi(\tau) + 2\pi\int_0^\tau \phi(\tau-\sigma)[\dot{\alpha}(\sigma) + \ddot{\xi}(\sigma) \\ &\quad + (0.5 - a_h)\ddot{\alpha}(\sigma)]\,\mathrm{d}\sigma \end{aligned} \tag{3}$$

$$\begin{aligned} C_M(\tau) &= \pi(0.5 + a_h)[\alpha(0) + \dot{\xi}(0) + (0.5 - a_h)\dot{\alpha}(0)]\phi(\tau) \\ &\quad + \pi(0.5 + a_h)\int_0^\tau \phi(\tau-\sigma)[\dot{\alpha}(\sigma) + \ddot{\xi}(\sigma) \\ &\quad + (0.5 - a_h)\ddot{\alpha}(\sigma)]\,\mathrm{d}\sigma + \frac{\pi}{2}a_h(\ddot{\xi} - a_h\ddot{\alpha}) \\ &\quad - (0.5 - a_h)\frac{\pi}{2}\ddot{\alpha} - \frac{\pi}{16}\ddot{\alpha} \end{aligned} \tag{4}$$

where the Wagner function $\phi(\tau)$ is given by

$$\phi(\tau) = 1 - \psi_1 e^{-\varepsilon_1\tau} - \psi_2 e^{-\varepsilon_2\tau} \tag{5}$$

The constants $\psi_1 = 0.165$, $\psi_2 = 0.335$, $\varepsilon_1 = 0.0455$, and $\varepsilon_2 = 0.3$ are quoted from [18], and $P(\tau)$ and $Q(\tau)$ are the externally applied forces and moments, respectively. We introduce four new variables,

$$w_1 = \int_0^\tau e^{-\varepsilon_1(t-\sigma)}\alpha(\sigma)\,\mathrm{d}\sigma; \qquad w_2 = \int_0^\tau e^{-\varepsilon_2(t-\sigma)}\alpha(\sigma)\,\mathrm{d}\sigma$$

$$w_3 = \int_0^\tau e^{-\varepsilon_1(t-\sigma)}\xi(\sigma)\,\mathrm{d}\sigma; \qquad w_4 = \int_0^\tau e^{-\varepsilon_2(t-\sigma)}\xi(\sigma)\,\mathrm{d}\sigma$$

and a variable vector $X = (x_1, x_2, \ldots, x_8)^T$ defined as $x_1 = \alpha$, $x_2 = \dot{\alpha}$, $x_3 = \xi$, $x_3 = \dot{\xi}$, $x_5 = w_1$, $x_6 = w_2$, $x_7 = w_3$, and $x_8 = w_4$. Then Eqs. (1) and (2) can be rewritten as a set of eight first-order ordinary differential equations [16]:

$$\dot{X} = f(X) \tag{6}$$

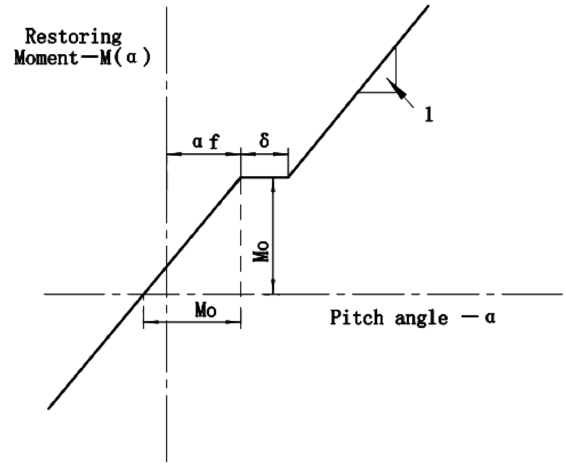

Fig. 2 Restoring moment due to preloaded free-play nonlinear stiffness.

Preloaded Free-Play Nonlinear Stiffness

The rotational stiffness of aircraft control surface often exhibits preloaded free-play nonlinearities. The rotation moment of control surface corresponds to the pitching moment of the airfoil. Figure 2 shows preloaded free-play nonlinear relationship between restoring moment and pitch angle. For this type of concentrated structural nonlinearity, the restoring moment can be expressed by a nonlinear function $M(\alpha)$:

$$M(\alpha) = \begin{cases} M_0 + \alpha - \alpha_f; & \alpha < \alpha_f \\ M_0; & \alpha_f \le \alpha \le \alpha_f + \delta \\ M_0 + \alpha - \alpha_f - \delta & \alpha_f < \alpha \end{cases} \tag{7}$$

Using Eq. (6), Eq. (7) can be integrated numerically by standard fourth-order Runge–Kutta scheme once the initial conditions$\alpha(0)$, $\dot{\alpha}(0)$, $\xi(0)$, and $\dot{\xi}(0)$ are given and then aeroelastic response of the airfoil with preloaded free-play nonlinear stiffness can be obtained.

Uncertainty Quantification

Uncertainties in Preloaded Free-Play Nonlinearity

In the corresponding deterministic structural and aerodynamic model, considering that the plunge DOF has linear stiffness and the pitching DOF has preloaded free-play nonlinear stiffness. The parameters of the system are $\mu = 100, \bar{\omega} = 0.2, r_\alpha = 0.5, x_\alpha = 0.25$, $a_h = -0.5$, $\zeta_\xi = \zeta_\alpha = 0$, and $a_f = 0.25°$. In the present study, assume that uncertainties exist in parameters M_0 and δ of preloaded free-play nonlinearity and also in initial pitching angle. The parametersM_0, δ and stochastic initial pitching angle are assumed to be Gaussian random variables as listed in Table 1.

Uncertainty Propagation and Probabilistic Sensitivity Analysis

The uncertainty propagation is the effect of variable uncertainties on the uncertainty of a function based on them. In this paper we research the effect of parametric uncertainties of M_0 and δ on aeroelastic response of the airfoil by MCS. By carrying out the deterministic analysis with the uncertain input parameters repeatedly, we can obtain enough responses of the airfoil as samples. Based

Table 1 Uncertain parameters and stochastic initial pitching angle

Variables	Mean	Standard deviation
M_0	0.25°	0.005
δ	0.5°	0.005
α_0	1°	0.08

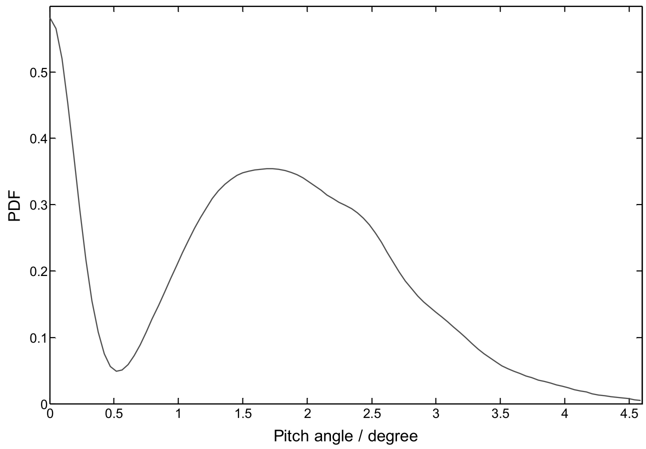

Fig. 3 PDF of pitching angle, $U^* = 5.5$.

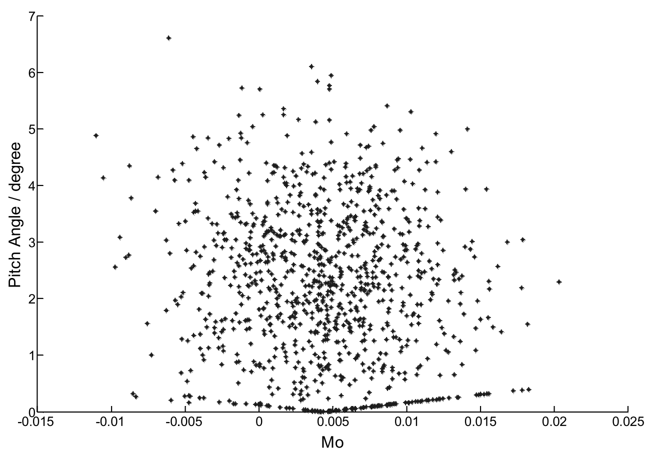

Fig. 4 Scatter plot of M_0 and pitching angle response.

on those samples we can get the probability density function (PDF) of response by kernel density estimation, a nonparametric estimation method [19]. The probability density is estimated by the following function:

$$f(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \tag{8}$$

where $x_1, \ldots, x_n$ is sample independent and identically distributed random variable of a random variable, K is the kernel (standard

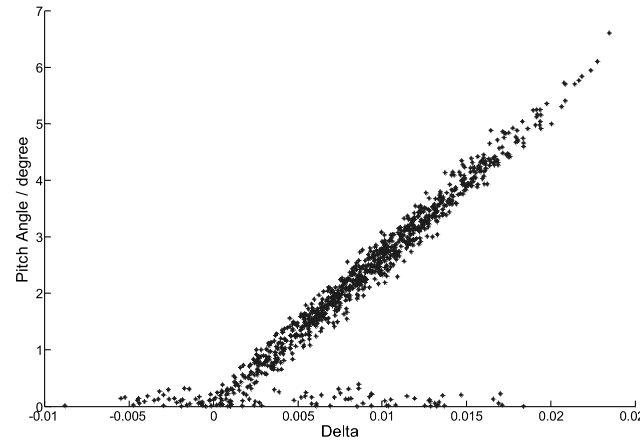

Fig. 5 Scatter plot of δ and pitching angle response.

Gaussian function here), and h is the bandwidth (smoothing parameter). Figure 3 shows the PDF of the pitching angle response of the airfoil.

There are two peaks in the curve in Fig. 3 and it can be explained as that the pitching angles of the airfoil distribute mainly in the neighborhood of 1.5–2 deg (unstable, LCO) and 0 deg (stable) when uncertain parameters and stochastic initial pitching angles exit.

To make clear which one of the uncertain parameters is the most important, probabilistic sensitivity analysis should be performed to simultaneously study the effect of parameters M_0 and δ on the response of the airfoil. The following scatter plot can be obtained by Monte Carlo simulation, in which the deterministic aeroelastic analysis was repeated 1000 times.

Figures 4 and 5 show that δ can influence the response (pitching angle) of the airfoil more intensively than that of M_0. Thus, it is reasonable just to take the uncertainty of δ into consideration in the following probabilistic aeroelastic analysis.

Probabilistic Aeroelastic Analysis

In this section, the aeroelastic analysis of the two-degree-of-freedom airfoil with the uncertain parameter δ and stochastic initial pitching angle was carried out repeatedly to get enough samples of aeroelastic responses. Based on those samples, the PDF and cumulative distribution function (CDF) of the responses of the airfoil (amplitude of the LCO) are obtained. By integrating CDF we can get the occurrence probability of the airfoil LCO or flutter, as shown in Fig. 6.

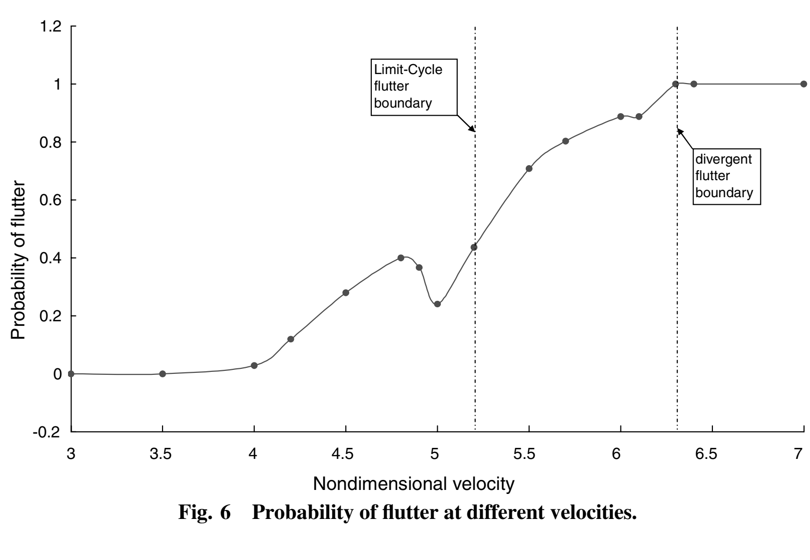

Fig. 6 Probability of flutter at different velocities.

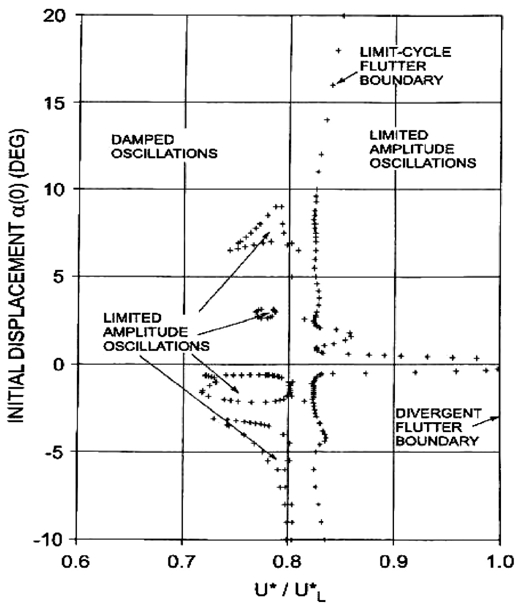

Fig. 7 Flutter boundary for the airfoil [16] ($\mu = 100$, $\bar{\omega} = 0.2$, $a_f = 0.25°$, $\delta = 0.5°$, and $M_0 = 0.25°$).

Results and Discussions

Figure 6 shows that flutter probability of the airfoil increases from 0 to 100% when velocity is near the divergent flutter boundary. Compared with the flutter boundary results of the same airfoil (shown in Fig. 7), which were given in [16], it is clear that we can obtain additional information about the stability of the airfoil with parametric uncertainties (probability of LCO) by uncertainty propagation and uncertainty quantification. In Fig. 7 there are some limited amplitude oscillation circular regions at the left range of limit cycle flutter boundary, which means that the probability of flutter is not zero at the velocity near the limit cycle flutter boundary. It can also be found that there is a limited blank range between the limited amplitude oscillation circular regions and the limit cycle flutter boundary in Fig. 7; it is the reason that the probability of flutter goes down at the velocities that are close to the limit cycle flutter boundary in Fig. 6.

Conclusions

In this paper we study the effect of parametric uncertainties on the responses of a two-degree-of-freedom airfoil oscillating with preloaded free-play nonlinear stiffness in pitch, and the probability of flutter (or probability of LCO) is obtained. Based on the analysis we demonstrate how to perform uncertainty propagation and uncertainty quantification analysis for aeroelastic system by the combined probabilistic method, Monte Carlo simulation, and nonparametric estimation.

The impact of uncertainties of parameters M_0 and δ in preloaded free-play nonlinearity on aeroelastic response of the airfoil is also studied. The scatter plot (Fig. 5) that is obtained by the probabilistic sensitive analysis shows that δ has a stronger influence on the amplitude of LCO than does M_0.

The propagation and quantification of uncertain parameter δ is analyzed using probabilistic method, the probability of flutter is obtained. Obviously, probabilistic aeroelastic analysis can give more information about the stability of aeroelastic system than conventional deterministic analysis.

References

[1] Pitt, D. M., Haudrich, D. P., and Thomas, M. J., Griffin, Kenneth E., “Probabilistic Aeroelastic Analysis and Its Implications on Flutter Margin Requirements,” AIAA Paper 2008-2198, 2008.

[2] Castravete, S. C., and Ibrahim, R. A., “Effect of Stiffness Uncertainties on the Flutter of a Cantilever Wing,” *AIAA Journal*, Vol. 46, No. 4, April 2008, pp. 925–935.
doi:10.2514/1.31692

[3] Ostenfeld-Rosenthal, P., Madsen, H. O., and Larsen, A., “Probabilistic Flutter Criteria for Long Span Bridges,” *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 42, Nos. 1–3, 1992, pp. 1265–1276.
doi:10.1016/0167-6105(92)90133-U

[4] Ge, Y. J., Xiang, H. F., and Tanaka, H., “Application of the Reliability Analysis Model to Bridge Flutter Under Extreme Winds,” *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 86, No. 2, 2000, pp. 155–167.
doi:10.1016/S0167-6105(00)00008-8

[5] Jakobsen, J. B., and Tanaka, H., “Modeling Uncertainties in Prediction of Aeroelastic Bridge Behavior,” *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 91, Nos. 12–15, 2003, pp. 1485–1498.
doi:10.1016/j.jweia.2003.09.006

[6] Liaw, D. G., and Yang, H. T. Y., “Reliability of Uncertain Laminated Shells Due to Buckling and Supersonic Flutter,” *AIAA Journal*, Vol. 29, No. 10, 1991, pp. 1698–1708.
doi:10.2514/3.10793

[7] Liaw, D. G., and Yang, H. T. Y., “Reliability of Initially Compressed Uncertain Laminated Plates in Supersonic Flow,” *AIAA Journal*, Vol. 29, No. 6, 1991, pp. 952–960.
doi:10.2514/3.10685

[8] Lindsley, N. J., Beran, P. S., and Pettit, C. L., “Effects of Uncertainty on the Aerothermoelastic Flutter Boundary of a Nonlinear Plate,” AIAA Paper 2002-5136, 2002.

[9] Lindsley, N. J., Beran, P. S., and Pettit, C. L., “Effects of Uncertainty on Nonlinear Plate Aeroelastic Response,” AIAA Paper 2002-1271, 2002.

[10] Kurdi, M., Lindsley, N., and Beran, P., “Uncertainty Quantification of Flutter Boundary in Goland Wing,” AIAA Atmospheric Flight Mechanics Conference and Exhibit, AIAA Paper 2007-6309, Hilton Head, SC, Aug. 2007.

[11] Heeg, J., “Stochastic Characterization of Flutter using Historical Wind Tunnel data,” 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, AIAA 2007-1769, Honolulu, HI, April 2007.

[12] Pettit, C. L., “Uncertainty Quantification in Aeroelasticity: Recent Results and Research Challenges,” *Journal of Aircraft*, Vol. 41, No. 5, Sept.–Oct. 2004, pp. 1217–1229.
doi:10.2514/1.3961

[13] Pettit, C. L., and Beran, P. S., “Polynomial Chaos Expansion Applied to Airfoil Limit-Cycle Oscillations,” *Collection of Technical Papers: AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, AIAA, Reston, VA, 2004, pp. 1975–1985.

[14] Attar, P. J., and Dowell, E. H., “Stochastic Analysis of the Limit-Cycle Behavior of a Nonlinear Aeroelastic Model Using the Response Surface Method,” 46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, AIAA Paper 2005-1986, 2005, pp. 2318–2338.

[15] Pettit, C. L., and Beran, P. S., “Effects of Parametric Uncertainty on Airfoil Limit Cycle Oscillation,” *Journal of Aircraft*, Vol. 40, No. 5, 2003, pp. 1004–1006.
doi:10.2514/2.6889

[16] Lee, B. H. K., Price, S. J., and Wong, Y. S., “Nonlinear Aeroelastic Analysis of the Airfoils: Bifurcation and Chaos,” *Progress in Aerospace Sciences*, Vol. 35, 1999, pp. 205–334.
doi:10.1016/S0376-0421(98)00015-3

[17] Fung Y. C., *An Introduction to the Theory of Aeroelasticity*, Dover, New York, 1993.

[18] Jones, R. T., “The Unsteady Lift of a Wing Of Finite Aspect Ratio,” NACA Rept 681, 1940.

[19] Xizhi, W., *Non-Parametric Estimation*, China Statistic Press, Beijing, 2006.